Voronoi Games

Frederik Brasz '09

FrankFest
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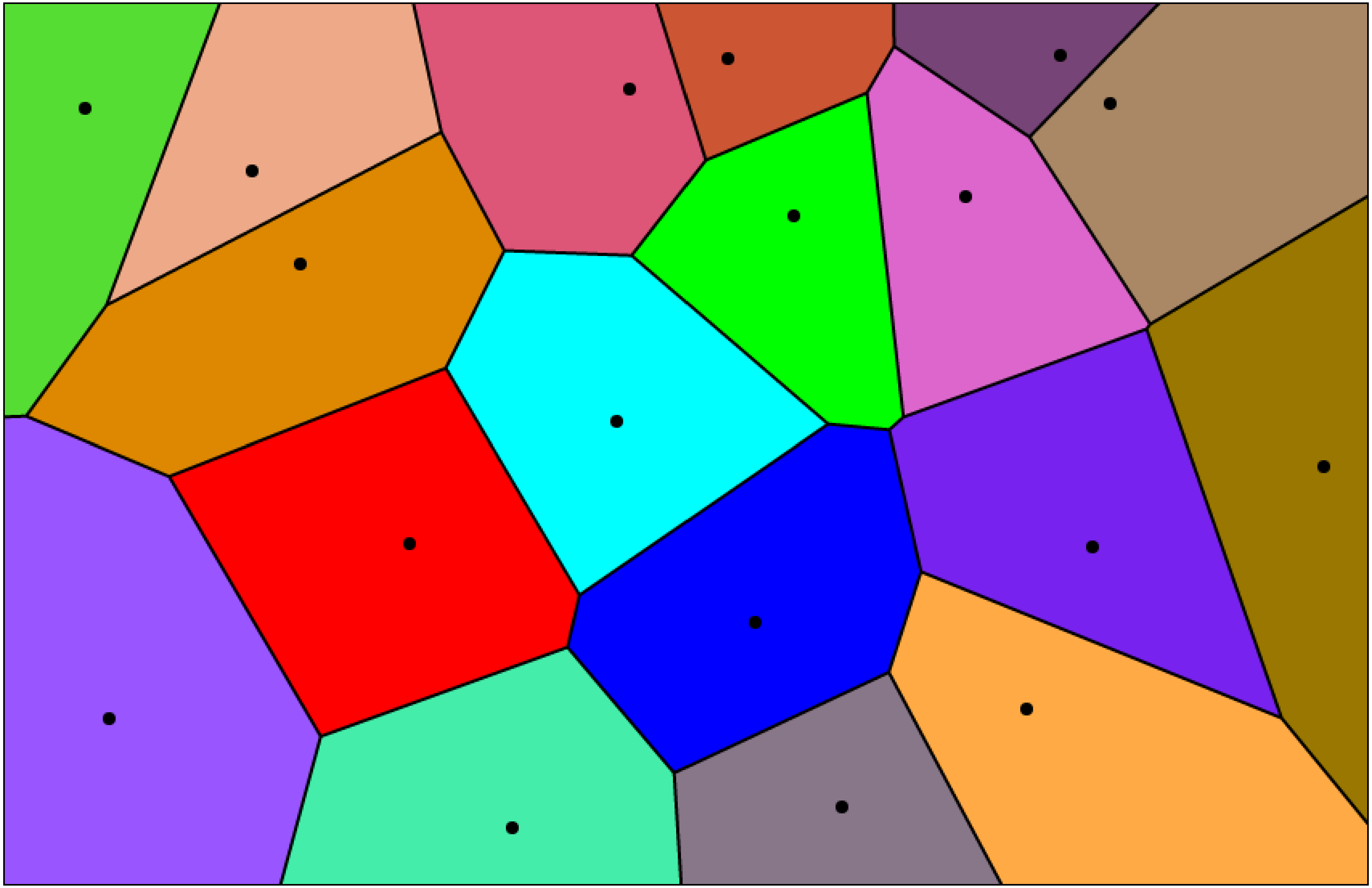
Outline

- Introduction to Voronoi diagrams
  - Applications
- The Voronoi game
  - Demo
  - Optimal strategy in 1D
  - Simulated results in 2D
- Extension: weighted Voronoi game

Voronoi diagrams



Voronoi diagrams

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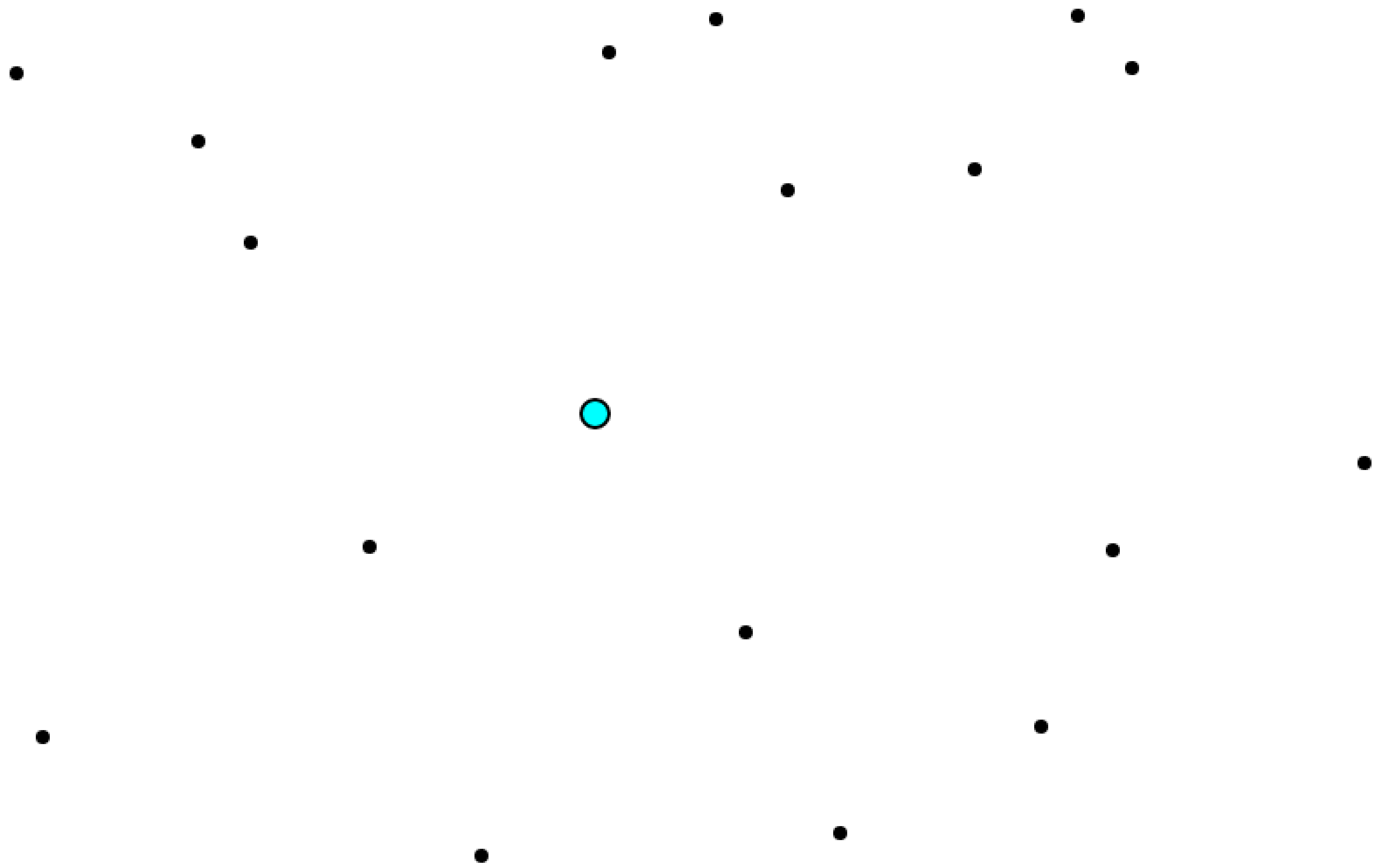

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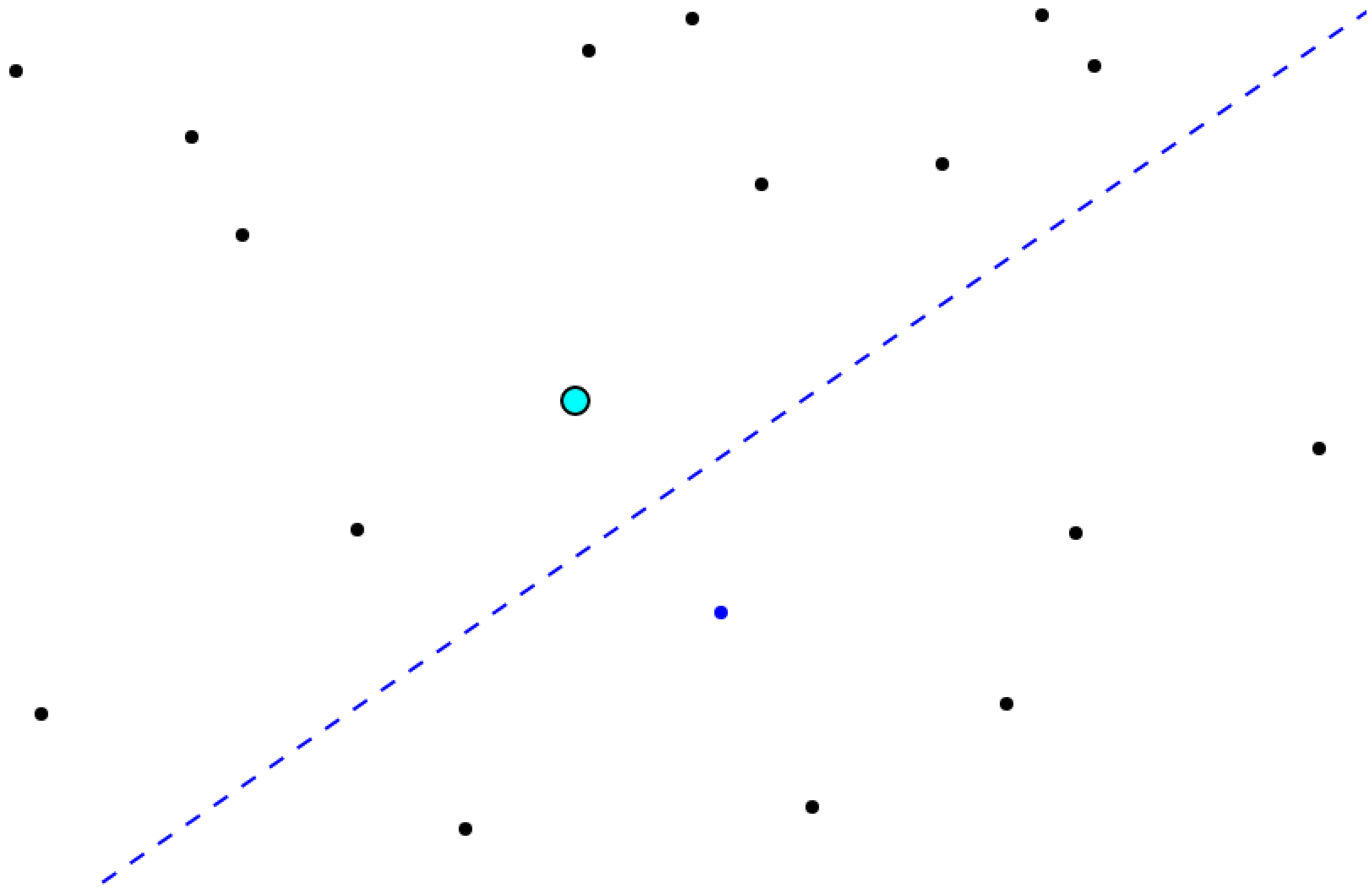

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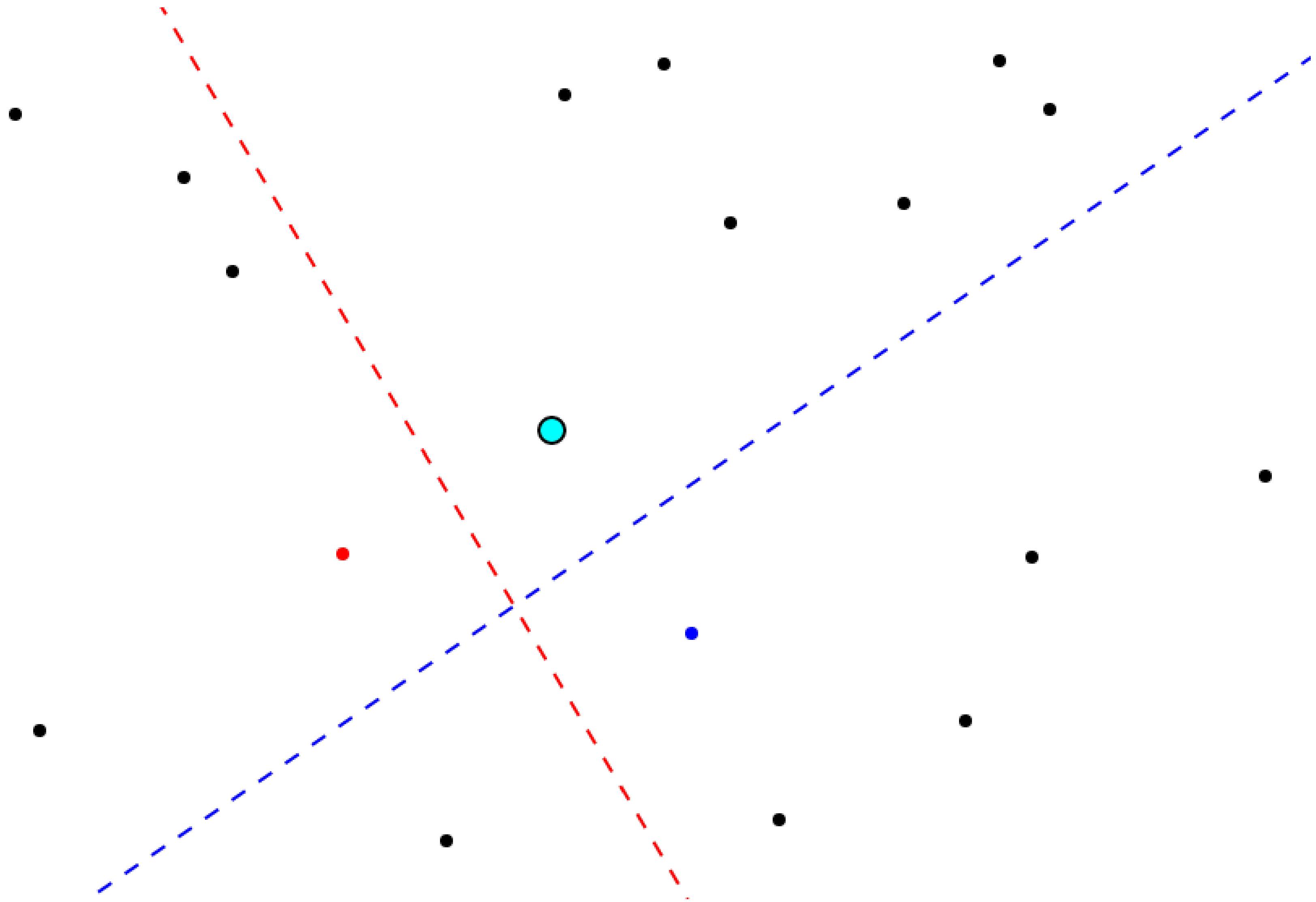

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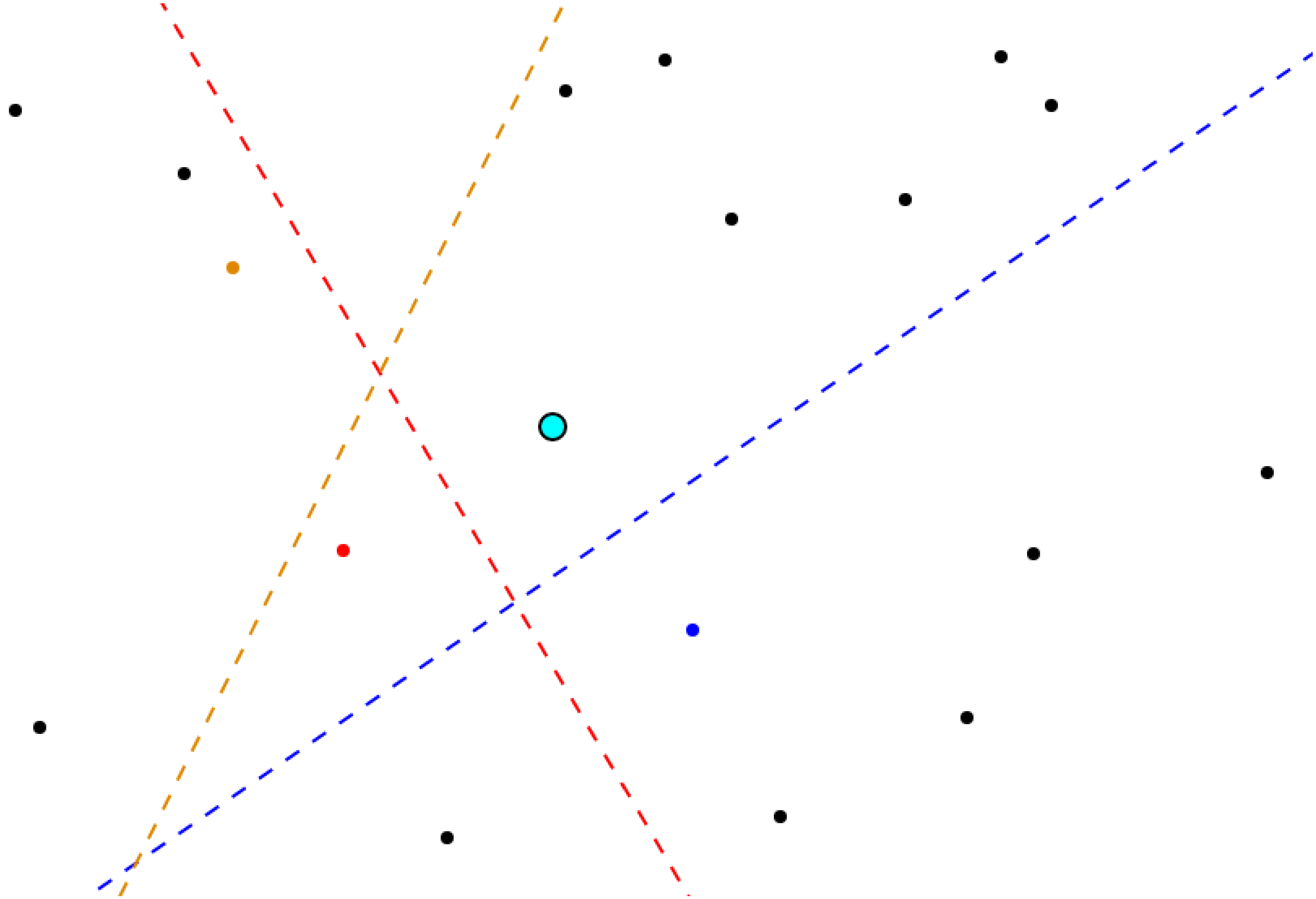

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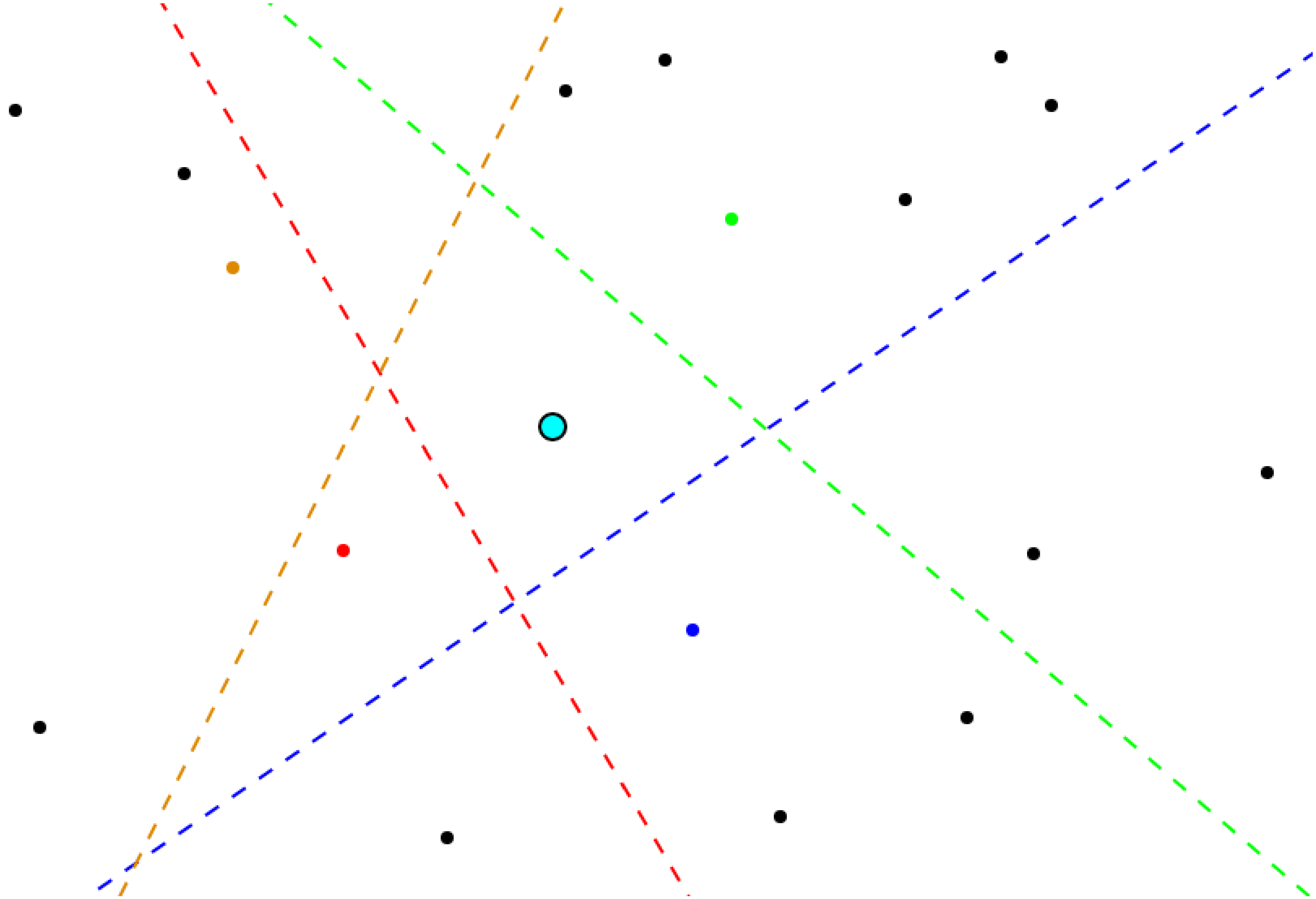

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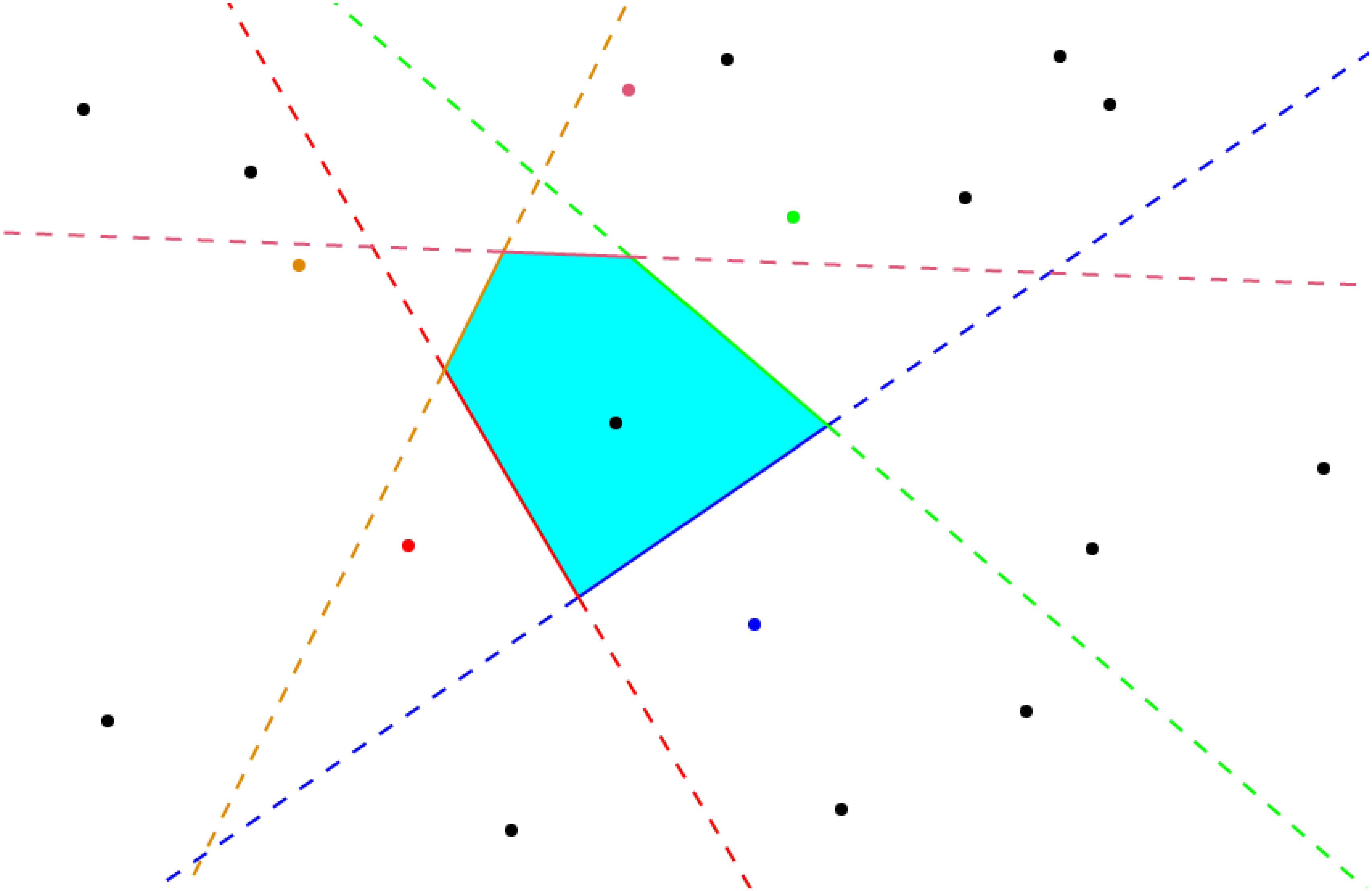

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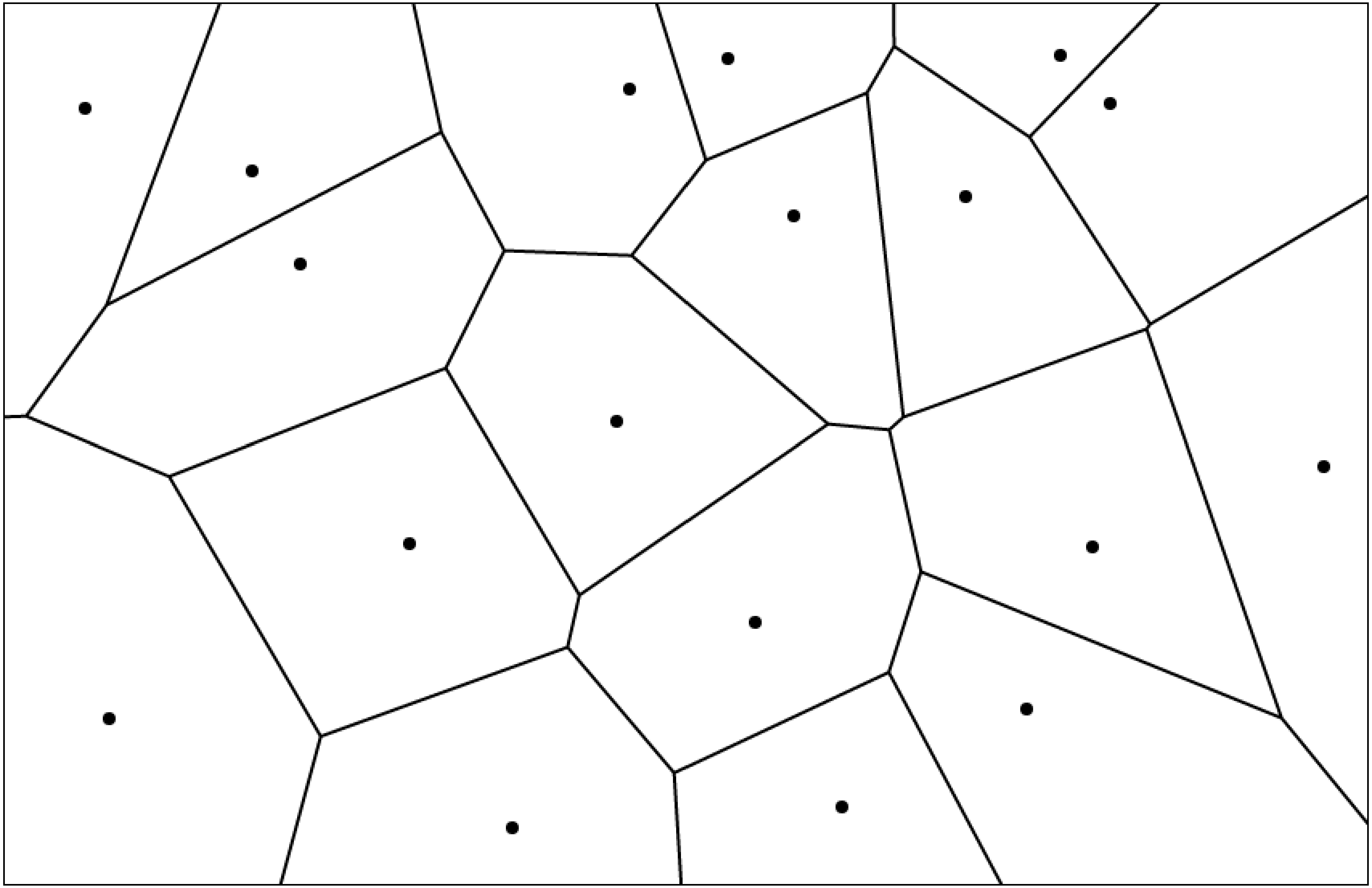

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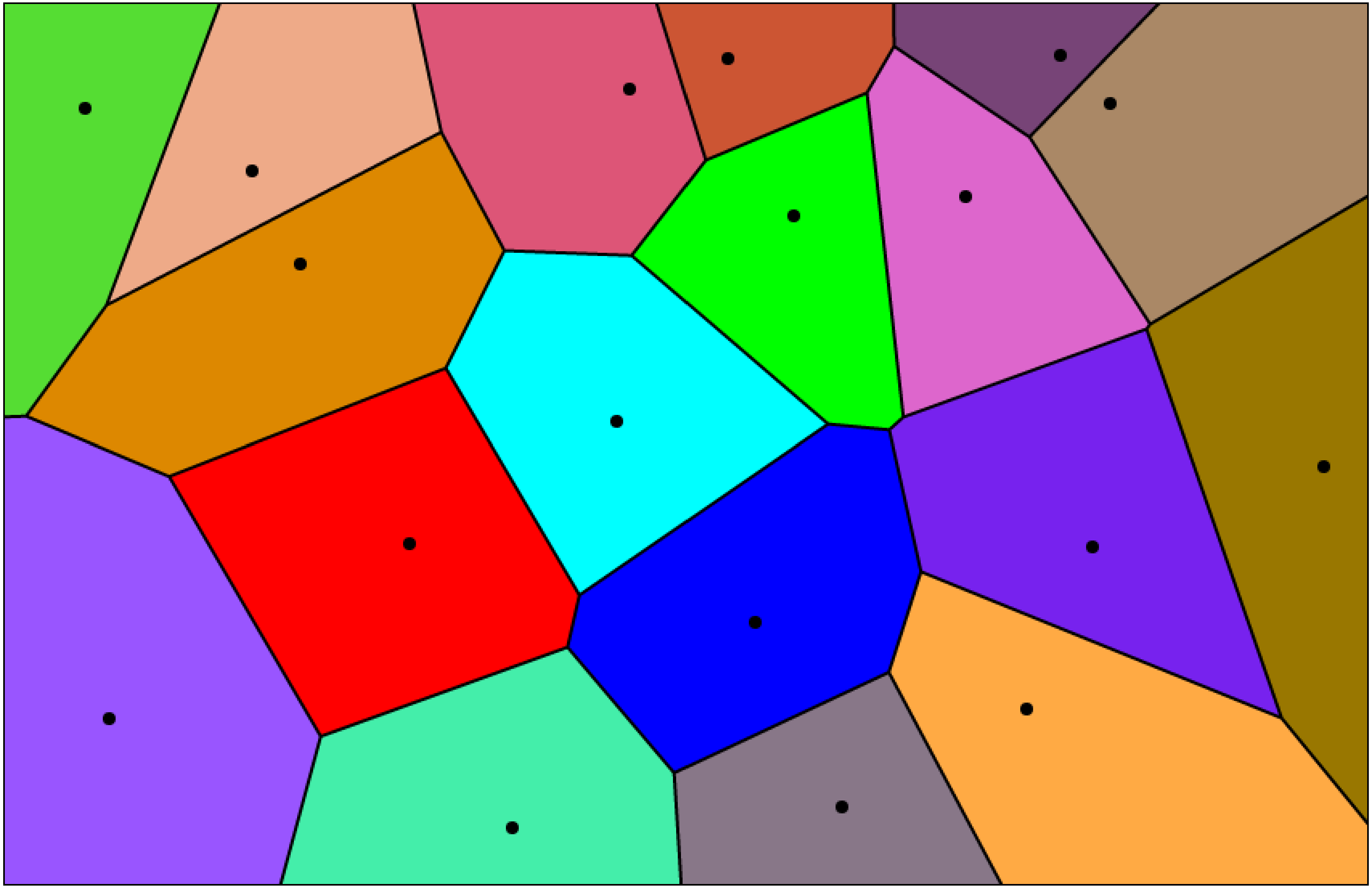

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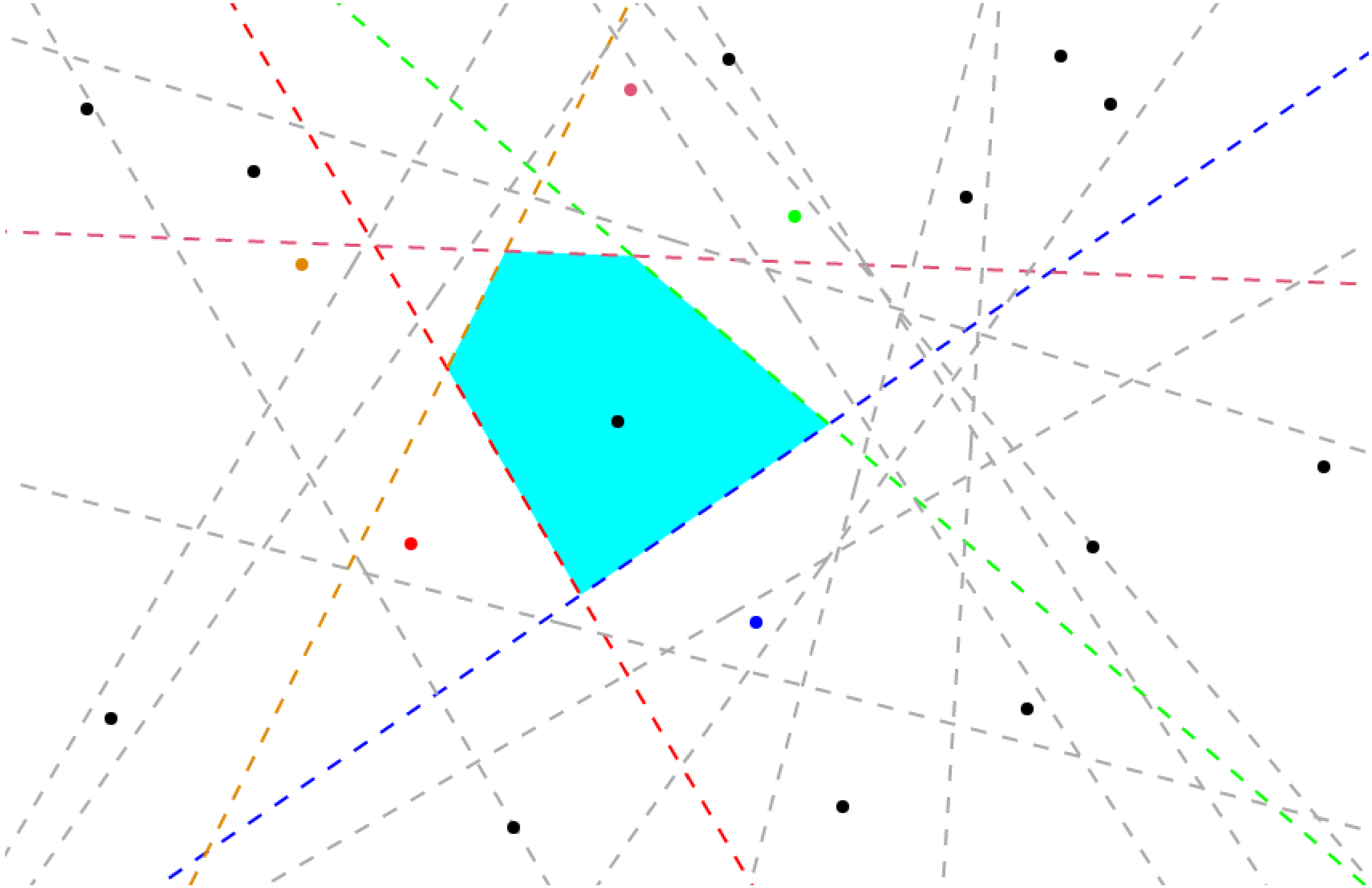
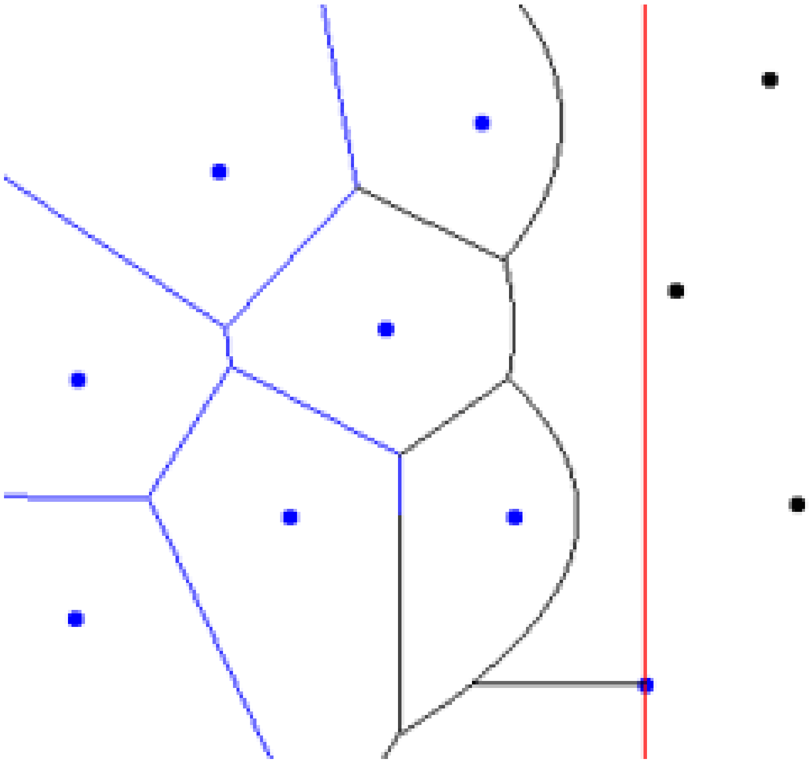


Fortune's sweep line algorithm



Source: https://en.wikipedia.org/wiki/Fortune%27s_algorithm

Computational complexity (steps required):

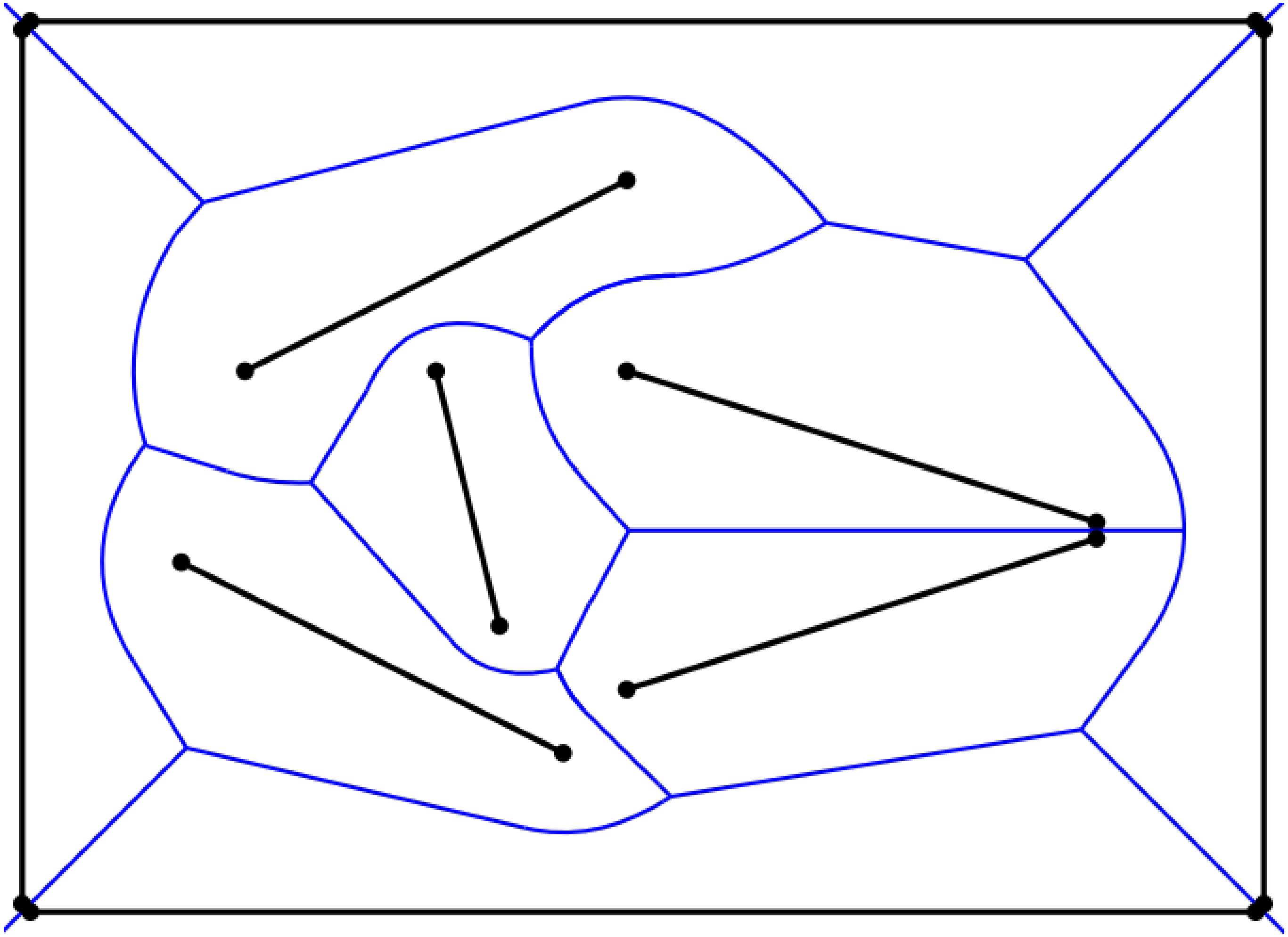
$$O(n \log n)$$

Steps for half-plane construction:

$$O(n^2)$$

Applications of Voronoi diagrams

- Robotics – route planning

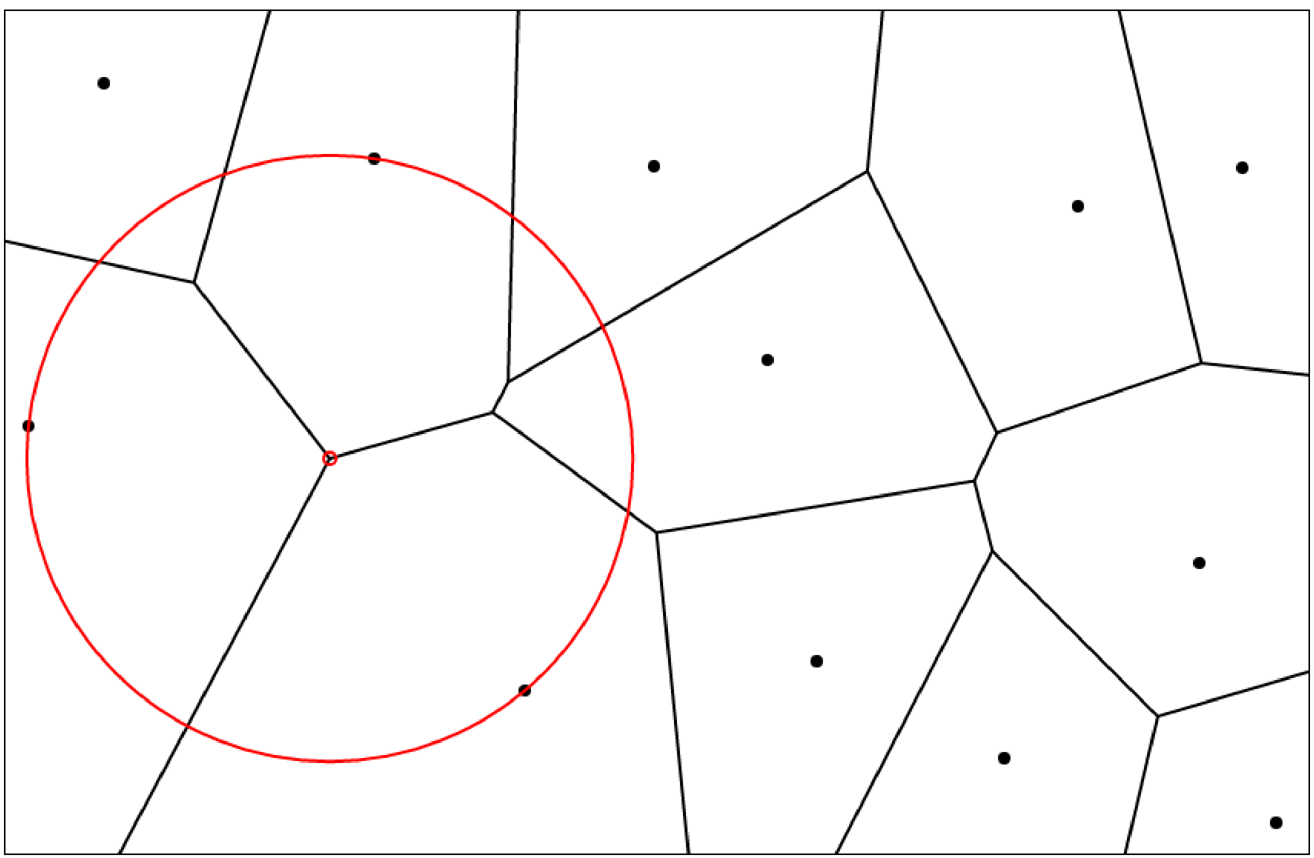


Source: http://www.cise.ufl.edu/~sitharam/COURSES/CG/kreveldmorevoronoi.pdf

Avoid obstacles by traversing edges of diagram

Applications of Voronoi diagrams

- Facility location
  - Place furthest from existing facilities
  - Find largest empty circle (vertex of diagram)

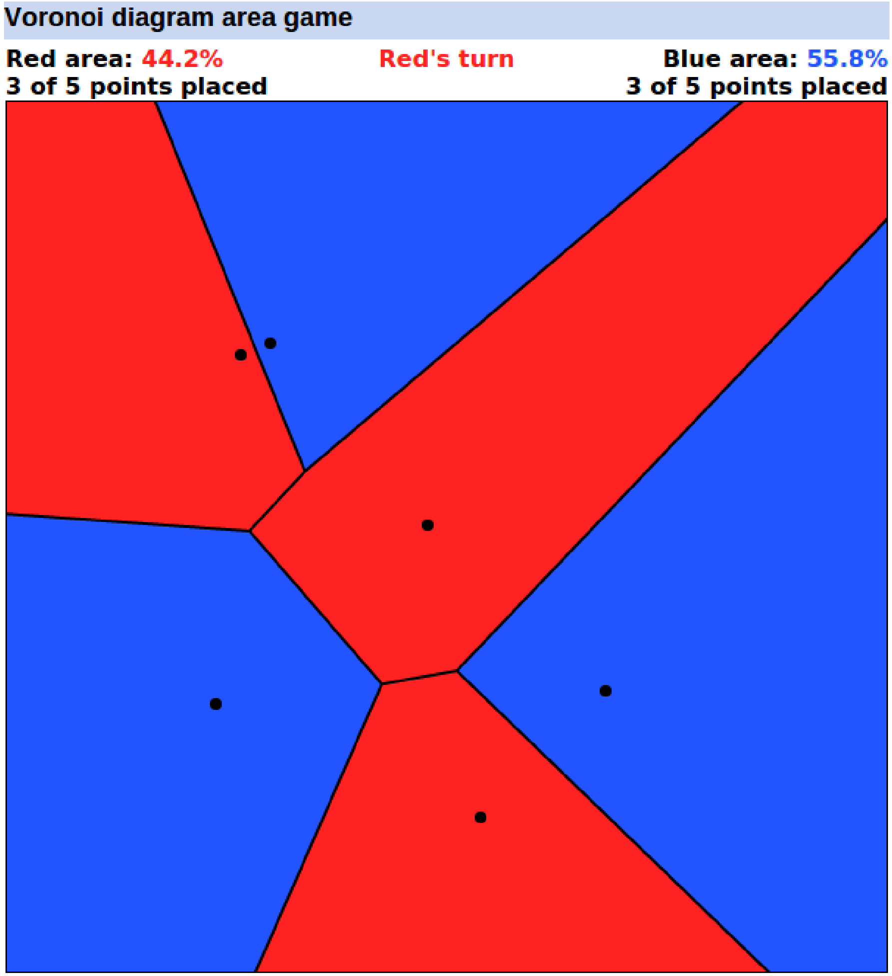


- Existing facilities
- Optimal location for new facility

The Voronoi game

Model for competitive facility location

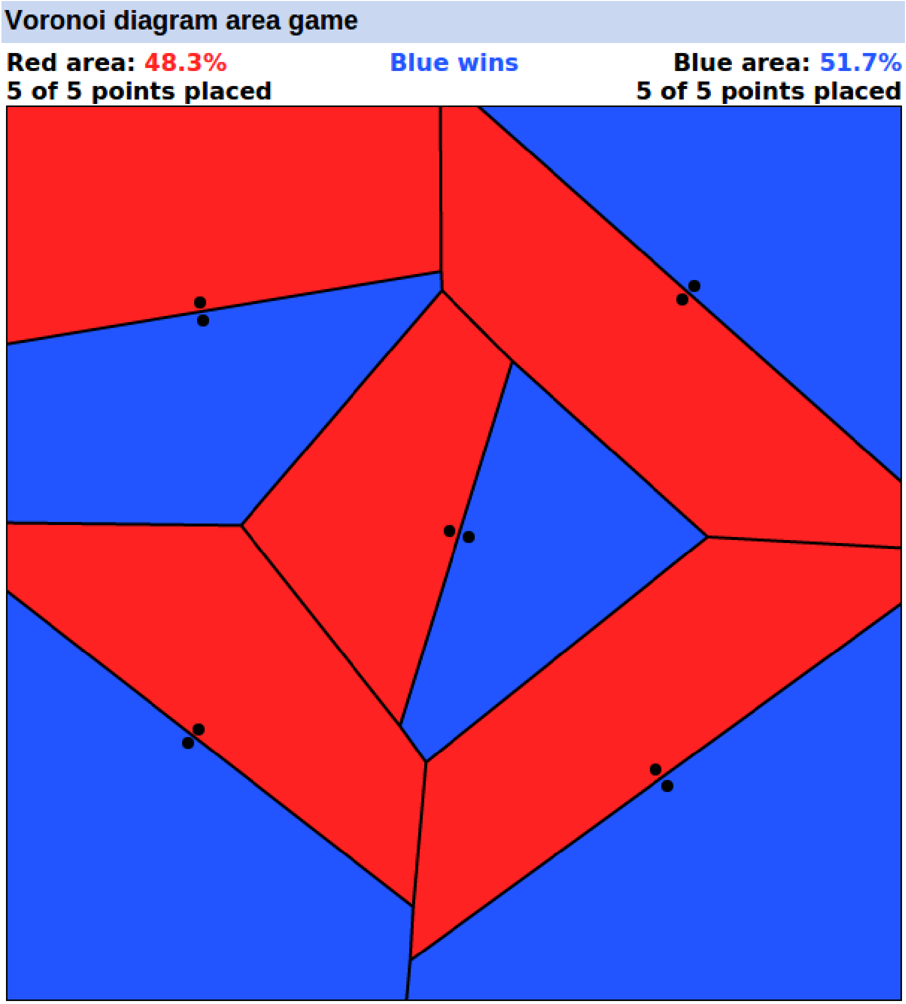
- Take turns placing sites
- Capture more area than opponent after *n* turns

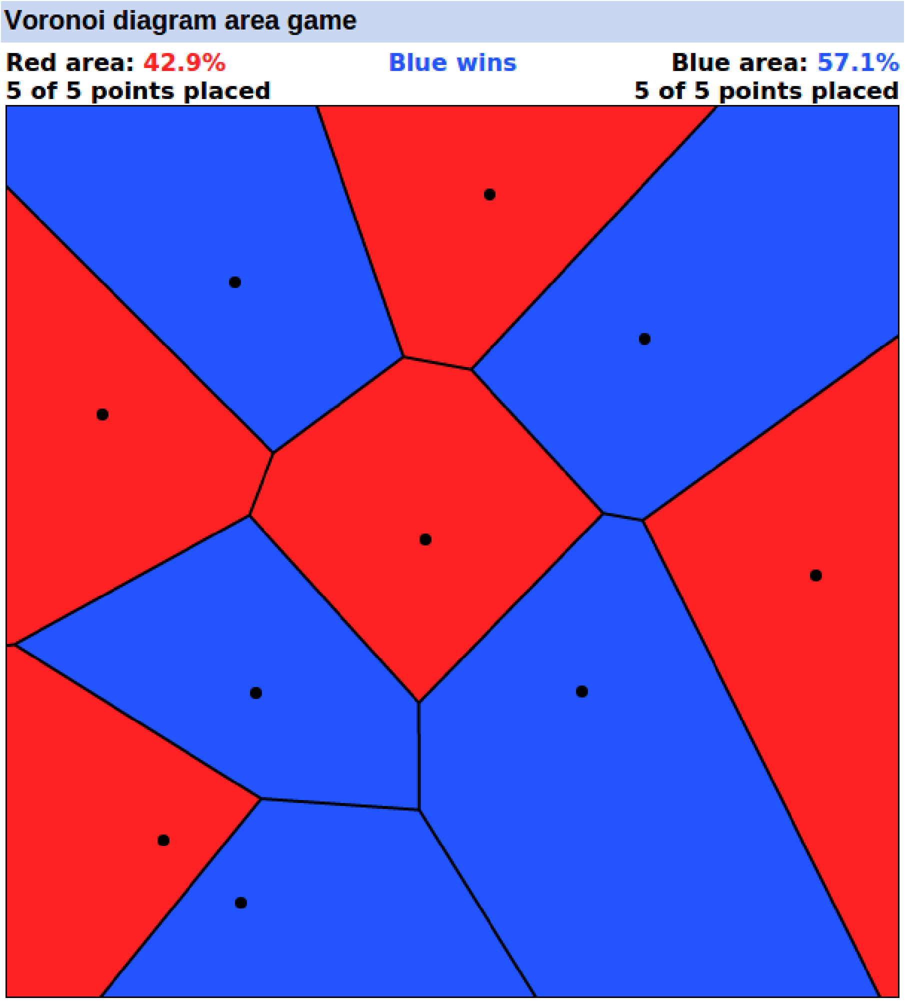


Play it at http://cfbrasz.github.io

The Voronoi game

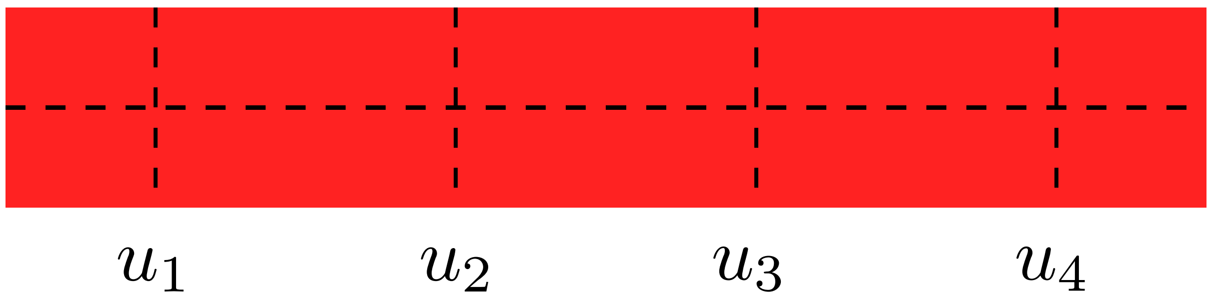
- Optimal strategies?
- Greedy algorithms or balanced cells?





The 1D Voronoi game

- Player 2 (blue) has winning strategy in 1D game



- Define *n* keypoints, given *n* turns per player:

$$u_i = (1 + 2i)/n$$

- Place sites on keypoints when available

Source: H. Ahn et al., Competitive facility location: the Voronoi game, *Theoretical Computer Science*, 310.1 (2004)

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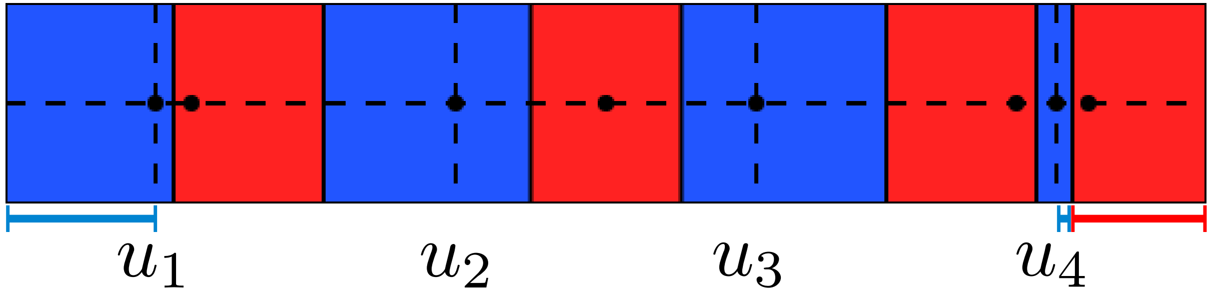

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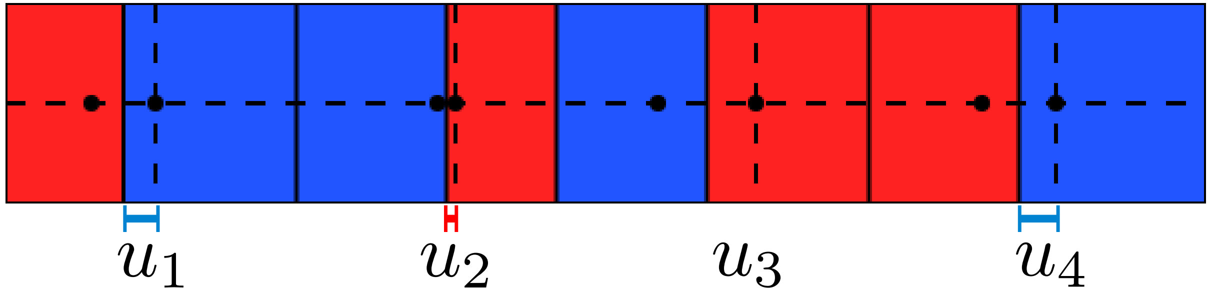

- Advantage of going last – can place always place closer to opponent's sites

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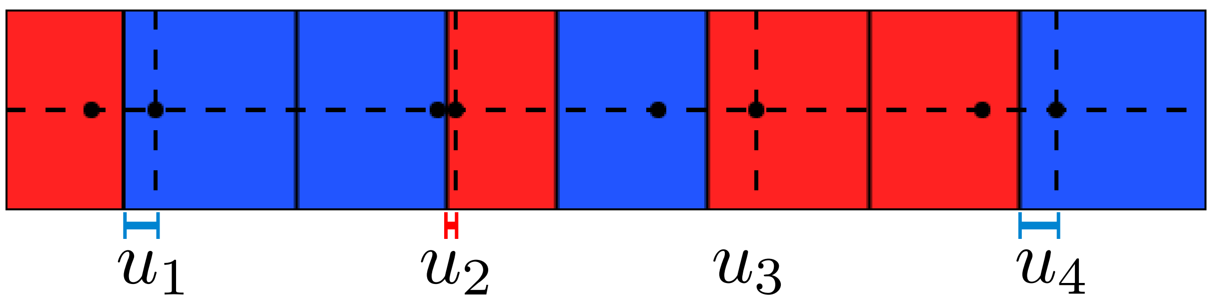

- Advantage of going last – can place always place closer to opponent's sites

Note: red can make blue only win by ε by placing within $\varepsilon/2n$ of the keypoints

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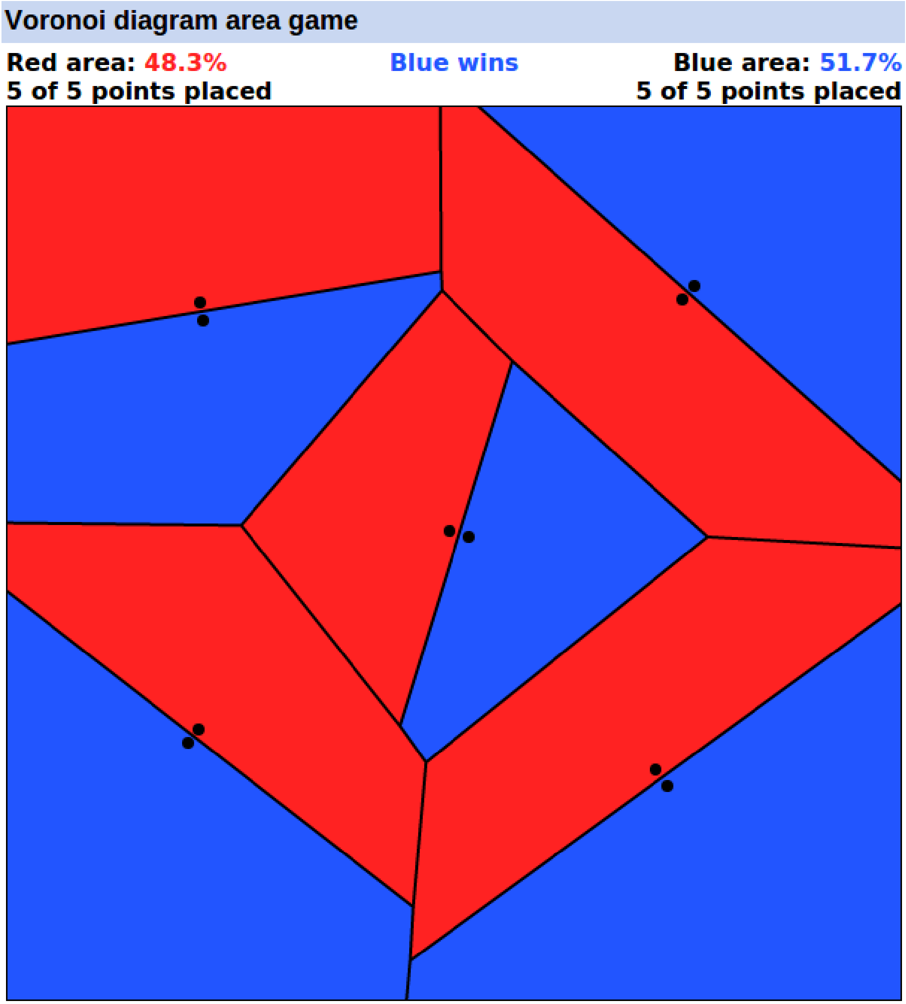
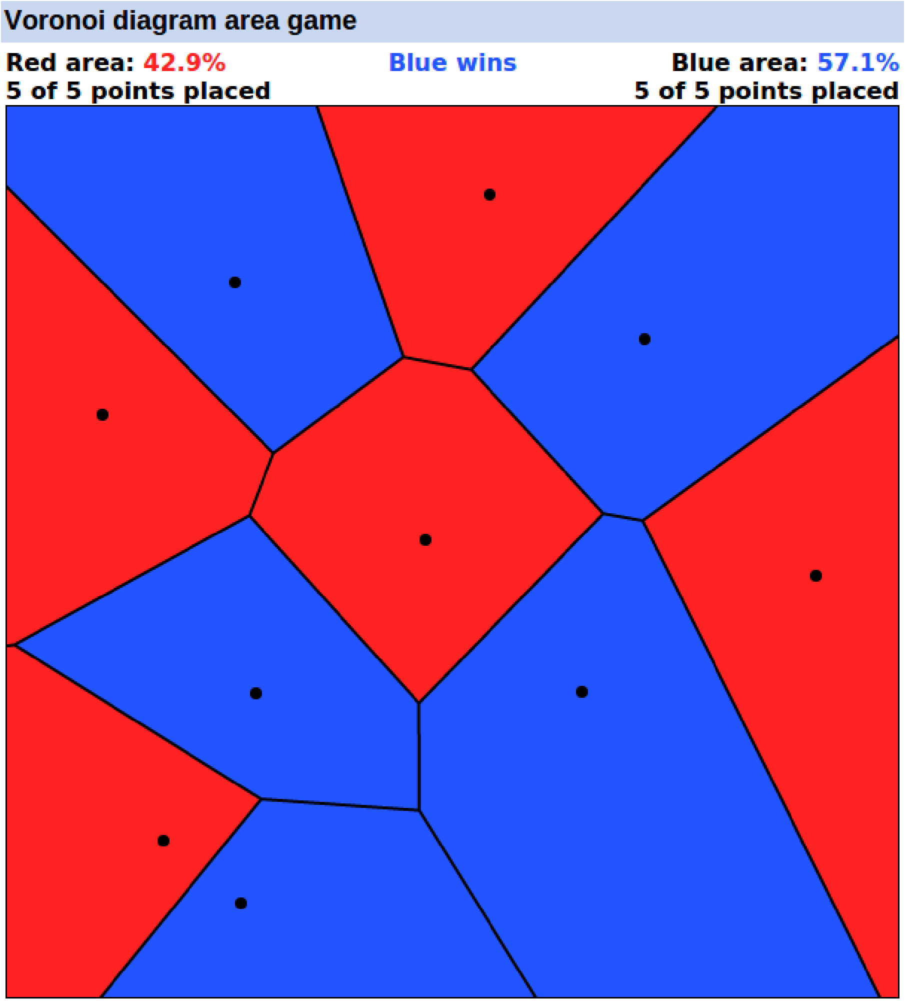
2D Voronoi game

- Optimal strategies? Can player 2 always win?
- Greedy algorithms or balanced cells?





2D Voronoi game

- No winning strategy known

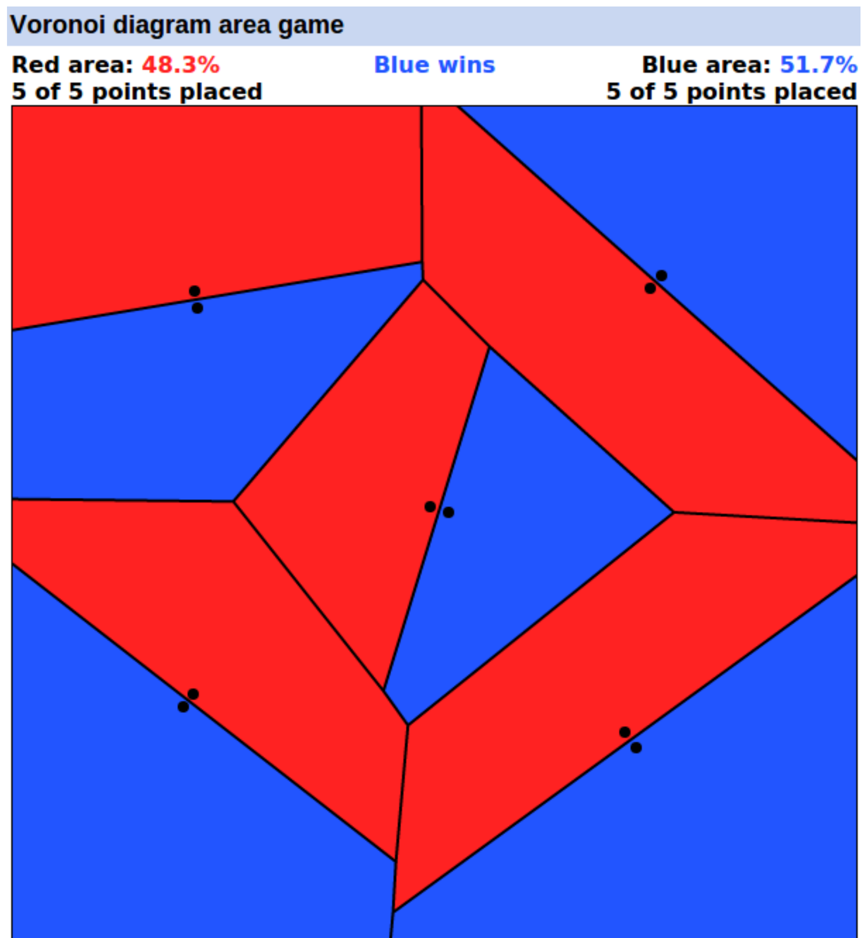
2D Voronoi game

- No winning strategy known
- Simulated games found 2nd player winning 85% of the time (with n=5 turns each)

Source: B. Bouzy et al., MCTS Experiments on the Voronoi Game, *Advances in Computer Games*, 7168 (2011)

2D Voronoi game

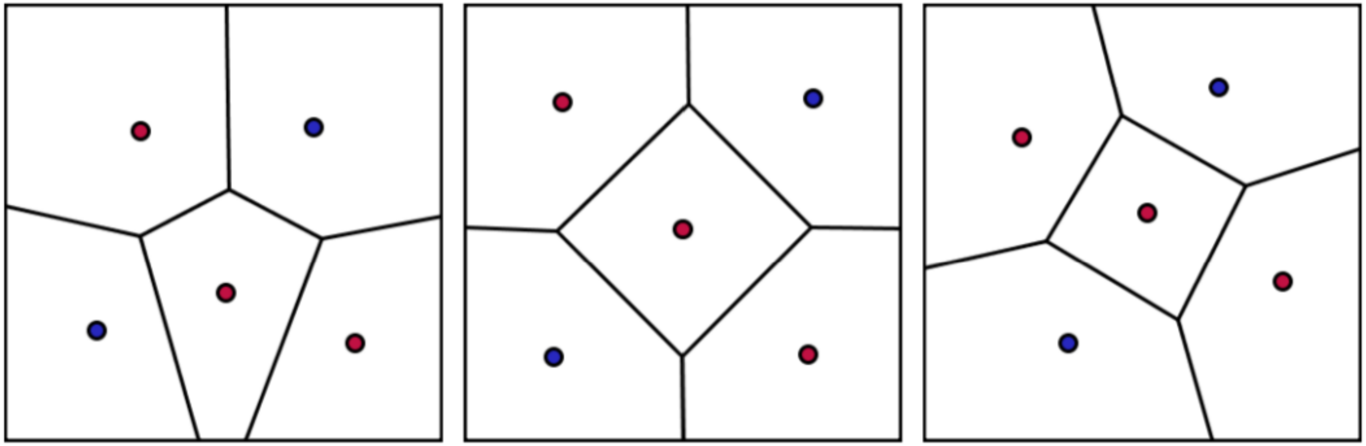
- Handicap around +5% (of area) given to 1st player to make game fair



Source: B. Bouzy et al., MCTS Experiments on the Voronoi Game, *Advances in Computer Games*, 7168 (2011)

2D Voronoi game

- Adding Voronoi knowledge to simulations improves win percentage:
  - Last-move depth-one search
  - Attacks on unbalanced cells



Balanced cells for *n*=5

Source: B. Bouzy et al., MCTS Experiments on the Voronoi Game, *Advances in Computer Games*, 7168 (2011)

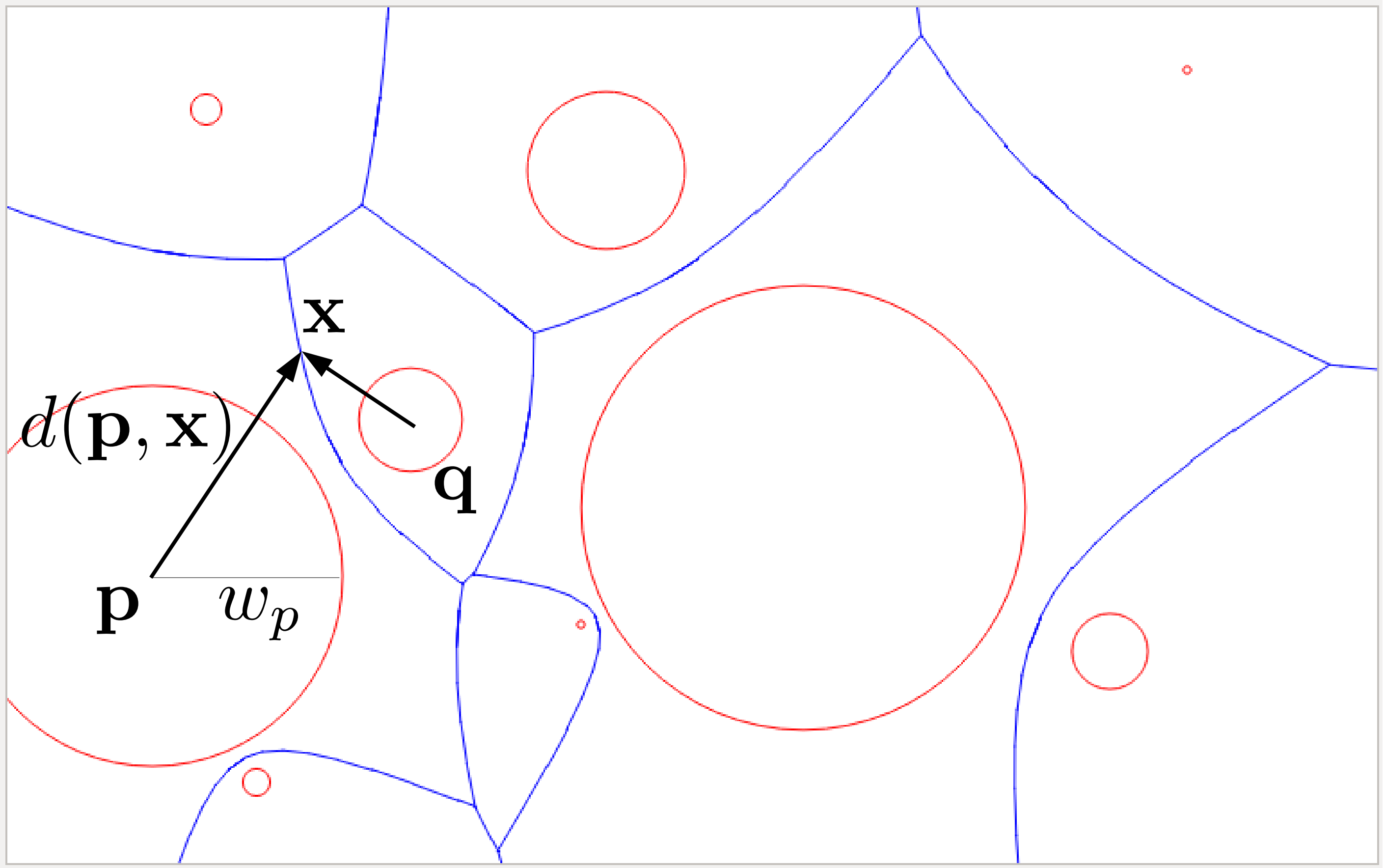
Additively weighted Voronoi diagrams

- Boundary between sites **p** and **q** defined by

$$d(\mathbf{p},\mathbf{x}) - w_p = d(\mathbf{q},\mathbf{x}) - w_q$$

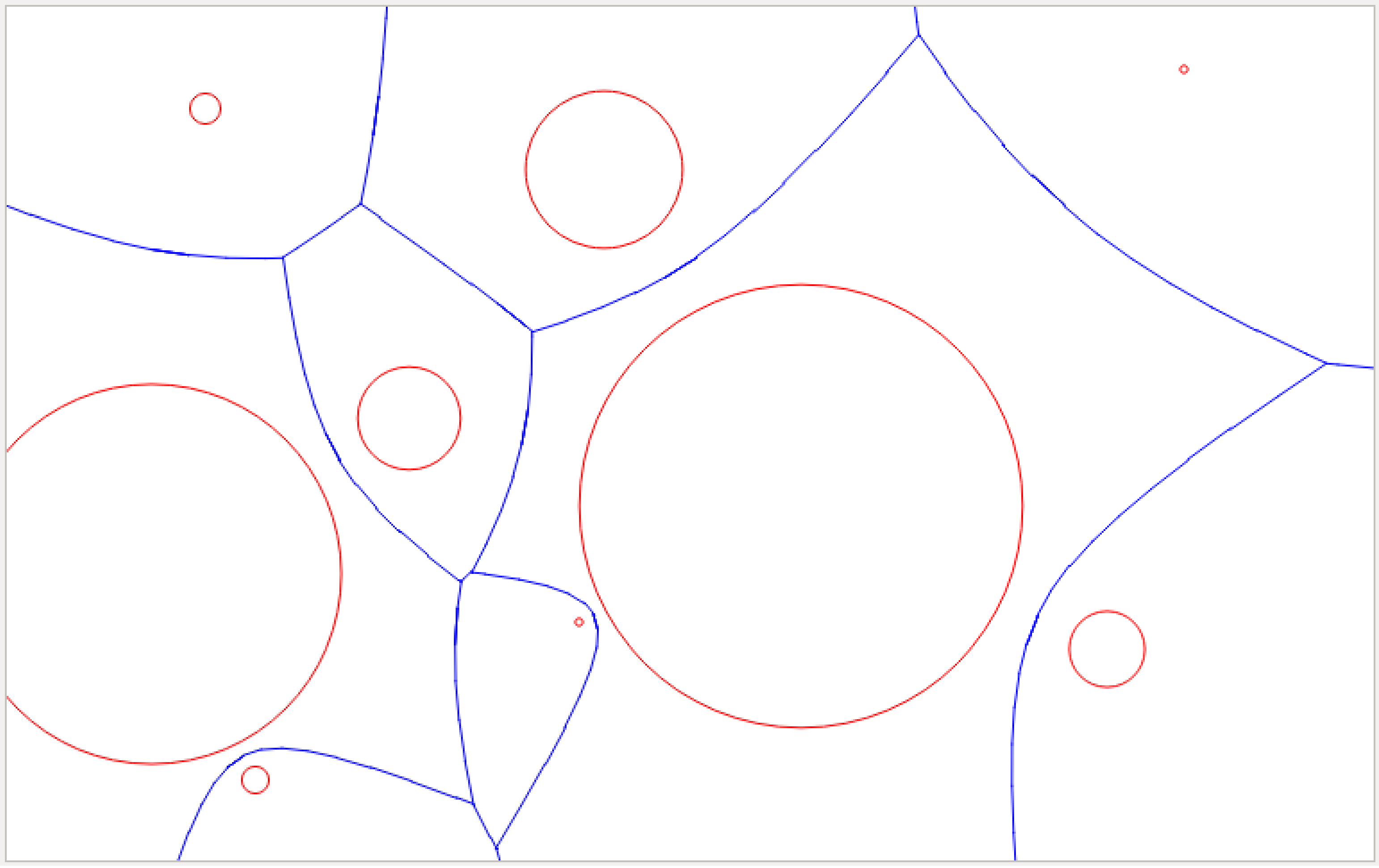
- Equivalent to Voronoi diagram with disks

Cell boundaries are segments of hyperbolas



Weighted Voronoi game?

On your turn, either place new site or add to weight of existing site



Viva la Discrete