Voronoi Games

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Outline

- Introduction to Voronoi diagrams
  - Applications
- The Voronoi game
  - Demo
  - Optimal strategy in 1D
  - Simulated results in 2D
- Extension: weighted Voronoi game
Voronoi diagrams
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Fortune's sweep line algorithm

Computational complexity (steps required):

\[ O(n \log n) \]

Steps for half-plane construction:

\[ O(n^2) \]
Applications of Voronoi diagrams

- Robotics – route planning

Avoid obstacles by traversing edges of diagram

Source: http://www.cise.ufl.edu/~sitharam/COURSES/CG/kreveldmorevoronoi.pdf
Applications of Voronoi diagrams

- **Facility location**
  - Place furthest from existing facilities
  - Find largest empty circle (vertex of diagram)
The Voronoi game

Model for competitive facility location

- Take turns placing sites
- Capture more area than opponent after \( n \) turns

Play it at http://cfbrasz.github.io
The Voronoi game

- Optimal strategies?
- Greedy algorithms or balanced cells?
The 1D Voronoi game

- Player 2 (blue) has winning strategy in 1D game

- Define $n$ keypoints, given $n$ turns per player:
  \[ u_i = \frac{(1 + 2i)}{n} \]

- Place sites on keypoints when available

The 1D Voronoi game

- Player 2 (blue) has winning strategy in 1D game

Define $n$ keypoints, given $n$ turns per player:

$$u_i = \frac{(1 + 2i)}{n}$$

- Place sites on keypoints when available

The 1D Voronoi game

- Player 2 (blue) has winning strategy in 1D game

- Advantage of going last – can place always closer to opponent's sites

The 1D Voronoi game

- Player 2 (blue) has winning strategy in 1D game

- Advantage of going last – can place always closer to opponent's sites

Note: red can make blue only win by $\varepsilon$ by placing within $\varepsilon/2n$ of the keypoints

2D Voronoi game

- Optimal strategies? Can player 2 always win?
- Greedy algorithms or balanced cells?

Voronoi diagram area game

<table>
<thead>
<tr>
<th>Red area</th>
<th>Blue wins</th>
<th>Blue area</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.3%</td>
<td>51.7%</td>
<td></td>
</tr>
</tbody>
</table>

Red area: 48.3%  
Blue area: 51.7%  
5 of 5 points placed

Voronoi diagram area game

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<tr>
<td>42.9%</td>
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Red area: 42.9%  
Blue area: 57.1%  
5 of 5 points placed
2D Voronoi game

- No winning strategy known
2D Voronoi game

- No winning strategy known
- Simulated games found 2\textsuperscript{nd} player winning 85\% of the time (with \(n=5\) turns each)

Source: B. Bouzy et al., MCTS Experiments on the Voronoi Game, Advances in Computer Games, 7168 (2011)
2D Voronoi game

- Handicap around +5% (of area) given to 1st player to make game fair

Source: B. Bouzy et al., MCTS Experiments on the Voronoi Game, Advances in Computer Games, 7168 (2011)
2D Voronoi game

- Adding Voronoi knowledge to simulations improves win percentage:
  - Last-move depth-one search
  - Attacks on unbalanced cells

Balanced cells for $n=5$

Source: B. Bouzy et al., MCTS Experiments on the Voronoi Game, Advances in Computer Games, 7168 (2011)
Additively weighted Voronoi diagrams

- Boundary between sites \( p \) and \( q \) defined by
  \[
  d(p, x) - w_p = d(q, x) - w_q
  \]

- Equivalent to Voronoi diagram with disks

Cell boundaries are segments of hyperbolas
Weighted Voronoi game?

On your turn, either place new site or add to weight of existing site
Viva la Discrete